SHOCK WAVE STRUCTURE IN A BINARY MIXTURE OF VISCOUS GASES

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Shock wave structure was studied in [1] using Struminskii's model [2] with the assumption that viscosity and thermal conductivity exist only as interactions between components. The present study will obtain asymptotic solutions of the problem of shock wave structure in the Navier–Stokes approximation.

1. The system of equations describing the flow of a binary gas mixture in the one-dimensional nonsteady state case has the form [2]

$$\frac{\partial \rho_i}{\partial t} + \frac{\partial \rho_i u_i}{\partial x} = 0, \quad \rho_i \frac{\partial u_i}{\partial t} + \rho_i u_i \frac{\partial u_i}{\partial x} + \frac{\partial p_i}{\partial x} = F_{ij} + \frac{\partial}{\partial x}\left(\mu_i \frac{\partial u_i}{\partial x}\right), \tag{1.1}$$

$$\rho_i \frac{\partial e_i}{\partial t} + \rho_i u_i \frac{\partial e_i}{\partial x} + p_i \frac{\partial u_i}{\partial x} = Q_{ij} + \mu_i \left(\frac{\partial u_i}{\partial x}\right)^2 + \frac{\partial}{\partial x}\left(\lambda_i \frac{\partial T_i}{\partial x}\right),$$

$$p_i = R_i \rho_i T_i, \; e_i = c_{iV} T_i, \; \rho_i = \rho_{ii} m_i, \; i = 1, 2, \; i \neq j,$$

where ρ_i, u_i, T_i, m_i, ρ_{ii} are the mean density, velocity, temperature, volume concentration, and true density of the i-th component. The quantities F_{ij} and Q_{ij} consider interaction between the components and are taken in the form

$$F_{ij} = K(u_j - u_i), \; Q_{ij} = K\varkappa_i (u_j - u_i)^2 + q(T_j - T_i),$$
$$\varkappa_1 + \varkappa_2 = 1.$$

We will assume that R_i, K, $\varkappa_i$, q, μ_i, λ_i, c_{iV} are some positive constants. In the future we will consider a mixture of monatomic gases, so that $\gamma_1 = \gamma_2 = \gamma$, where $\gamma_i = 1 + R_i/c_{iV}$.

Assuming that all the unknown functions of system (1.1) depend on $\xi = x - Dt$, where D is the shock wave velocity, we obtain

$$\rho_i V_i = c_i, \quad c_1 V_1 + c_2 V_2 + \frac{R_1 c_1 T_1}{V_1} + \frac{R_2 c_2 T_2}{V_2} = c_3 + \mu_1 \frac{dV_1}{d\xi} + \mu_2 \frac{dV_2}{d\xi}, \tag{1.2}$$

$$c_1\left(c_{1V} T_1 + \frac{V_1^2}{2}\right) + c_2\left(c_{2V} T_2 + \frac{V_2^2}{2}\right) + R_1 c_1 T_1 + R_2 c_2 T_2 = \mu_1 V_1 \frac{dV_1}{d\xi} +$$
$$+ \mu_2 V_2 \frac{dV_2}{d\xi} + \lambda_1 \frac{dT_1}{d\xi} + \lambda_2 \frac{dT_2}{d\xi} + c_4,$$

$$c_1 \frac{dV_1}{d\xi} + R_1 c_1 \frac{dT_1/V_1}{d\xi} = K(V_2 - V_1) + \mu_1 \frac{d^2 V_1}{d\xi^2},$$

$$c_1 c_{1V} \frac{dT_1}{d\xi} + \frac{R_1 c_1 T_1}{V_1}\frac{dV_1}{d\xi} = K\varkappa_1 (V_2 - V_1)^2 + q(T_2 - T_1) + \mu_1 \left(\frac{dV_1}{d\xi}\right)^2 + \lambda_1 \frac{d^2 T_1}{d\xi^2},$$

where c_i are integration constants and $V_i = u_i - D$.

We introduce dimensionless variables as follows:

$$\bar{V}_i = \frac{c_1 + c_2}{c_3} V_i, \quad \bar{T}_i = \frac{(R_1 c_1 + R_2 c_2)(c_1 + c_2)}{c_3^2} T_i, \quad \bar{\rho}_i = \frac{c_3}{(c_1 + c_2)^2} \rho_i, \tag{1.3}$$

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$$\bar{\mu}_i = \frac{\mu_i}{\mu_*}, \quad \bar{\lambda}_i = \frac{\lambda_i}{\lambda_*}, \quad \bar{\xi} = \frac{c_1 + c_2}{\mu_*}\xi, \quad \lambda_* = \frac{R_1 c_1 + R_2 c_2}{c_1 + c_2}\mu_*,$$
$$\bar{K} = \frac{K\mu_*}{(c_1 + c_2)^2}, \quad \bar{q} = \frac{\mu_*}{(c_1 + c_2)(R_1 c_1 + R_2 c_2)} q,$$

where μ_* is the dedimensioning viscosity factor, which may, in particular, coincide with the mixture viscosity ahead of the shock wave.

Substituting Eq. (1.3) in Eq. (1.2) and dropping the bar above the dimensionless quantities, we find

$$\rho_i V_i = \alpha_i^0, \quad \alpha_2^0 V_1 + \alpha_1^0 V_2 + m_1^0 \frac{T_1}{V_1} + m_2^0 \frac{T_2}{V_2} = 1 + \mu_1 \frac{dV_1}{d\xi} + \mu_2 \frac{dV_2}{d\xi}, \tag{1.4}$$
$$\frac{\gamma}{\gamma - 1} m_1^0 T_1 + \frac{\gamma}{\gamma - 1} m_2^0 T_2 + \frac{\alpha_1^0}{2} V_1^2 + \frac{\alpha_2^0}{2} V_2^2 = A + \mu_1 V_1 \frac{dV_1}{d\xi} + \mu_2 V_2 \frac{dV_2}{d\xi} +$$
$$+ \lambda_1 \frac{dT_1}{d\xi} + \lambda_2 \frac{dT_2}{d\xi},$$
$$\alpha_1^0 \frac{dV_1}{d\xi} + m_1^0 \frac{d(T_1/V_1)}{d\xi} = K(V_2 - V_1) + \mu_1 \frac{d^2 V_1}{d\xi^2},$$
$$\frac{m_1^0}{\gamma - 1} \frac{dT_1}{d\xi} + m_1^0 \frac{T_1}{V_1} \frac{dV_1}{d\xi} = K\varkappa_1 (V_2 - V_1)^2 + q(T_2 - T_1) + \mu_1 \left(\frac{dV_1}{d\xi}\right)^2 + \lambda_1 \frac{d^2 T_1}{d\xi^2},$$

where $A = c_4(c_1 + c_2)/c_3^2$, $\alpha_i^0 = c_i/(c_1 + c_2)$ $(i = 1, 2)$ is the mass concentration of the i-th mixture component ahead of the shock wave.

We pose the following boundary problem for system (1.4): to find a solution $V_i(\xi)$, $T_i(\xi)$ of system (1.4) which will tend to a constant value at infinity, i.e., as $\xi \to -\infty$,

$$V_i \to V_i^0, \quad T_i \to T_i^0, \quad dV_i/d\xi \to 0, \quad dT_i/d\xi \to 0, \tag{1.5}$$

as $\xi \to +\infty$,

$$V_i \to V_i^1, \quad T_i \to T_i^1, \quad dV_i/d\xi \to 0, \quad dT_i/d\xi \to 0.$$

The necessary condition for the existence of such a solution is obviously the requirement that $V_i = V_i^0$, $T_i = T_i^0$ and $V_i = V_i^1$, $T_i = T_i^1$ be equilibrium positions of system (1.4). This will occur if $V_1^0 = V_2^0 = V^0$, $T_1^0 = T_2^0 = T^0 = V^0(1 - V^0)$, $V_1^1 = V_2^1 = V^1 = 2\gamma/(\gamma + 1) - V^0$, $T_1^1 = T_2^1 = T^1 = V^1(1 - V^1)$, $V^0 = \rho_0 D^2/(\rho_0 D^2 + p_0)$, where ρ_0, p_0 are the mixture density and pressure ahead of the shock wave. It can easily be proved that the points (V^0, T^0), (V^1, T^1) exist and are unique by directly solving system (1.4) with all derivatives with respect to ξ set equal to zero. For the future we will assume that thermal conductivity coefficients may be neglected, i.e., $\lambda_1 = 0$, $\lambda_2 = 0$.

2. We will consider the case where strong velocity interaction exists, i.e., $1/K \ll 1$. With this assumption in the zeroth approximation we may rewrite Eq. (1.4) in the form

$$V_1 = V_2 = V, \quad V + \frac{1}{V}(m_1^0 T_1 + m_2^0 T_2) = 1 + \mu \frac{dV}{d\xi}, \tag{2.1}$$
$$\frac{\gamma}{\gamma - 1}(m_1^0 T_1 + m_2^0 T_2) + \frac{1}{2} V^2 = A + \mu V \frac{dV}{d\xi},$$
$$\frac{m_1^0}{\gamma - 1} \frac{dT_1}{d\xi} + m_1^0 \frac{T_1}{V} \frac{dV}{d\xi} = q(T_2 - T_1) + \mu_1 \left(\frac{dV}{d\xi}\right)^2, \quad \mu = \mu_1 + \mu_2.$$

Integrating system (2.1) with the condition that $V(0) = (V^0 + V^1)/2$, we obtain

$$\xi = \frac{2\mu V^0}{(V^0 - V^1)(\gamma + 1)} \ln(V^0 - V) - \frac{2\mu V^1}{(V^0 - V^1)(\gamma + 1)} \ln(V - V^1) - \frac{2\mu}{\gamma + 1} \ln \frac{V^0 - V^1}{2}; \tag{2.2}$$
$$T_2 - T_1 = \eta \left(\frac{V^2}{\gamma + 1} - \frac{2}{\gamma + 1} V + \frac{V^0 - V^1}{\gamma - 1}\right) - \eta\theta V^{-(\gamma - 1)} e^{-\theta\xi} \int_{-\infty}^{\xi} e^{\theta\tau} f(V)\, d\tau; \tag{2.3}$$

$$T_1 = V(1-V) + \mu V \frac{dV}{d\xi} - m_2^0 (T_2 - T_1), \tag{2.4}$$

where

$$f(V) = V^{(\gamma-1)}\left[\frac{V^2}{\gamma+1} - \frac{2}{\gamma+1}V + \frac{V^0 - V^1}{\gamma-1}\right], \quad \theta = \frac{q(\gamma-1)}{m_1^0 m_2^0},$$

Using Eq. (2.2), we write Eq. (2.3) in the form

$$T_2 - T_1 = \eta V^{-(\gamma-1)} \int_V^{V^0} u^{(\gamma-2)} \left(\frac{V^0-u}{V^0-V}\right)^\beta \left(\frac{V-V^1}{u-V^1}\right)^\alpha (V^0-u)(u-V^1)\,du, \tag{2.5}$$

where

$$\eta = (m_1^0\mu - \mu_1)(\gamma^2-1)/(2m_1^0 m_2^0 \mu).$$

Equations (2.2)-(2.4) define a solution of system (2.1) satisfying boundary conditions (1.5), as may be verified by direct transition to the limits as $V \to V^0$ or $V \to V^1$. In the general case the integral in Eq. (2.5) may be expressed in terms of hypergeometric functions of two variables. It is evident from Eq. (2.5) that at $\eta = 0$ or $m_1^0\mu_2 = m_2^0\mu_1$ $T_2 = T_1$, while at $\eta \gtrless 0$ $T_2 \gtrless T_1$. We will consider the case $\eta > 0$. It follows from the last equation of Eq. (2.1) that $dT_1/d\xi > 0$, i.e., $T_1(\xi)$ is a monotonically increasing function at $-\infty < \xi < +\infty$. At small $(V^0 - V)$ is can easily be shown from Eqs. (2.4), (2.5) that $T_2 - T^0 \sim -K_1(V^0 - V)$, while at small $(V - V^1)$ $T_2 - T^1 \sim -K_1(V - V^1)$, if $\alpha > 1$, and $T_2 - T^1 \sim K_2(V - V^1)^\alpha$, if $\alpha < 1$, where K_1, K_2 are positive constants. Consequently at $\alpha < 1$ $dT_2/d\xi$ changes sign. Figure 1 qualitatively shows the possible behaviors of the functions $T_2(\xi)$, $T_1(\xi)$, and $V(\xi)$. We will consider the limiting functions $T_i(\xi)$ and $V(\xi)$ as $\mu_i \to 0$. Taking $\mu_1/\mu_2 = k$, in the limit we obtain

$$V(\xi) = \begin{cases} V^0, & \xi < 0, \\ (V^0 + V^1)/2, & \xi = 0, \\ V^1, & \xi > 0, \end{cases} \quad T_1(\xi) = \begin{cases} T_0, & \xi < 0, \\ \varphi_1(\xi), & \xi > 0, \end{cases} \quad T_2(\xi) = \begin{cases} T^0, & \xi < 0, \\ \varphi_2(\xi), & \xi > 0, \end{cases}$$

$$\varphi_1(\xi) = T^1 - \frac{m_1^0 - km_2^0}{m_1^0(k+1)} (V^1)^{-(\gamma-1)} e^{-\theta\xi} \left[(V^1)^\gamma (1 - V^1) - (V^0)^\gamma (1 - V^0)\right],$$

$$\varphi_2(\xi) = 2T^1 - \varphi_1(\xi).$$

It follows from the expressions obtained for $\eta > 0$ that $T^0 < T_1^* = \varphi_1(0) < T^1$, $T_2^* = \varphi_2(0) > T^1 > T^0$. The limiting behavior of $T_2(\xi)$, $T_1(\xi)$, and $V(\xi)$ is shown in Fig. 1b. When $\eta < 0$ it is necessary to interchange T_2 and T_1 in Fig. 1. Thus in the limit as $\mu_i \to 0$ the solution has a discontinuity with subsequent continuous temperature relaxation zone, with the size of the discontinuity depending on the ratio $\mu_1/\mu_2 = k$.

3. We will consider the case of intense heat exchange between the components, i.e., $1/q \ll 1$. The formulation of the last equation of system (1.4) then simplifies and will have the form $T_1 = T_2 = T$. Moreover, we will assume that $m_1^0 \sim 0$, $\alpha_1^0 \sim m_1^0$, $\mu_1 \sim m_1^0$ for $\mu_1 \ll m_1^0$, $K \sim m_1^0$. If $K \gg m_1^0$, we obtain the intense velocity interaction considered in Section 2. Considering these approximations and dropping terms of higher order smallness, we transform Eq. (1.4) to the form

$$V_2 + \frac{T}{V_2} = 1 + \mu_2 \frac{dV_2}{d\xi}, \quad \frac{V_2^2}{2} + \frac{\gamma}{\gamma-1}T = A + \mu_2 V_2 \frac{dV_2}{d\xi}, \tag{3.1}$$

$$\sigma\frac{dV_1}{d\xi} + \frac{d(T_1/V_1)}{d\xi} = \tilde{K}(V_2 - V_1) + \tilde{\mu}_1 \frac{d^2V_1}{d\xi^2}, \quad \sigma = \frac{\alpha_1^0}{m_1^0}, \quad \tilde{K} = \frac{K}{m_1^0}, \quad \tilde{\mu}_1 = \frac{\mu_1}{m_1^0}.$$

Integrating the first two equations of system (3.1), we find

$$T = V_2(1 - V_2) - [(\gamma+1)/2](V^0 - V_2)(V_2 - V^1),$$

while $V_2(\xi)$ is defined, like $V(\xi)$ of Section 2, by Eq. (2.2), with μ replaced by μ_2. We will seek a solution of the last equation of system (3.1) for $V_1(\xi)$ with the assumption that $\varepsilon = V^0 - V^1$ is small, i.e., assuming a weak shock wave. To do this we introduce new dimensionless velocities v_i and temperatures τ_i defined by

$$V_i = \frac{\gamma}{\gamma+1} + \frac{\varepsilon}{2} v_i, \quad T_i = \frac{\gamma}{(\gamma+1)^2} + \frac{\gamma-1}{2(\gamma+1)}\varepsilon\tau_i - \frac{\varepsilon^2}{4}. \tag{3.2}$$

Now $V_i = V^0$, $T_i = T^0$ correspond to $v_i = 1$, $\tau_i = -1$, while $V_i = V^1$, $T_i = T^1 - v_i = -1$, $\tau_i = 1$. As a result, in the zeroth approximation in ε for v_1, V_2 and $\tau_1 = \tau_2 = \tau$ we will have

$$\frac{(\gamma+1)^2}{2\gamma\mu_2}\varepsilon\xi = \ln\frac{1-v_2}{1+v_2}, \quad \tau = -v_2, \tag{3.3}$$

$$\tilde{\mu}_1\frac{d^2v_1}{d\xi^2} - \frac{\gamma\sigma-1}{\gamma}\frac{dv}{d\xi} + \frac{\gamma-1}{\gamma}\frac{dv_2}{d\xi} + \tilde{K}(v_2 - v_1) = 0.$$

Integrating the last equation of system (3.3) we find

$$v_1 = \frac{1}{\tilde{\mu}_1(\nu_1-\nu_2)}\left\{\left(\tilde{K} + \frac{\gamma-1}{\gamma}\nu_1\right)e^{\nu_1\xi}\int_{\xi}^{+\infty} v_2 e^{-\nu_1 t}dt + \left(\tilde{K} + \frac{\gamma-1}{\gamma}\nu_2\right)e^{\nu_2\xi}\int_{-\infty} v_2 e^{-\nu_2 t}dt\right\}, \tag{3.4}$$

where ν_i are roots of the quadratic equation $\tilde{\mu}_1\nu^2 + [(1-\gamma\sigma)/\gamma]\nu - \tilde{K} = 0$, while $\nu_1 > 0$, $\nu_2 < 0$. The integrals appearing in Eq. (3.4) are expressible in terms of hypergeometric functions. Transforming from the expressions obtained to the limit as $\mu_2 \to 0$ with finite $\tilde{\mu}_1$, we have

$$v_2 = \begin{cases} 1, & \xi < 0, \\ 0, & \xi = 0, \\ -1, & \xi > 0, \end{cases} \quad v_1 = \begin{cases} 1 - 2\dfrac{\tilde{K}\gamma + (\gamma-1)\nu_1}{\gamma\tilde{\mu}_1\nu_1(\nu_1-\nu_2)}e^{\nu_1\xi}, & \xi < 0, \\ -1 - 2\dfrac{\tilde{K}\gamma + (\gamma-1)\nu_2}{\gamma\tilde{\mu}_1\nu_2(\nu_1-\nu_2)}e^{\nu_2\xi}, & \xi > 0, \end{cases} \tag{3.5}$$

$$\tau = -v_2.$$

Moreover, it can easily be shown from Eq. (3.5) that $\lim_{\xi\to-0} v_1 = v_1^* = \lim_{\xi\to+0} v_1$, and $\lim_{\xi\to-0}\frac{dv_1}{d\xi} \neq \lim_{\xi\to+0}\frac{dv_1}{d\xi}$, where $v_1^* = 1 - 2\frac{\tilde{K}\gamma + (\gamma-1)\nu_1}{\gamma\tilde{\mu}_1\nu_1(\nu_1-\nu_2)}$. The qualitative form of the functions of Eq. (3.5) is shown in Fig. 2a, where curves 1-3 correspond to possible behaviors of the function $v_i(\xi)$: 1) $\gamma\sigma > 2\gamma - 1$, 2) $\gamma\sigma < 2\gamma - 1$, 3) $\tilde{\mu}_1\tilde{K}\gamma^2 + \gamma(\sigma-1)(\gamma-1) < 0$. If we let $\mu_i \to 0$ $(i = 1, 2)$ we then obtain for v_1

$$v_1 = \begin{cases} 1, & \xi < 0, \\ -1 + \dfrac{2\gamma(1-\sigma)}{1-\gamma\sigma}e^{\nu_2\xi}, & \xi > 0 \end{cases} \quad \text{for } \gamma\sigma > 1,$$

$$v_1 = v_2 \quad \text{for } \gamma\sigma = 1, \quad v_1 = \begin{cases} 1 - 2\dfrac{\gamma(1-\sigma)}{1-\gamma\sigma}e^{\nu_1\xi}, & \xi < 0, \\ -1, & \xi > 0 \end{cases}$$

for $\gamma\sigma < 1$. In Fig. 2b curves 1-4 show the function $v_1(\xi)$ as $\mu_i \to 0$ for the cases $\gamma\sigma > 2\gamma - 1$, $\gamma < \gamma\sigma < 2\gamma - 1$, $1 < \gamma\sigma < \gamma$, and $\gamma\sigma < 1$. Thus the shock wave appears as a discontinuity with subsequent continuous relaxation zone at $\gamma\sigma > 1$, while for $\gamma\sigma < 1$ the continous solution ends in a discontinuity.

We will consider $\tilde{\mu}_1 \sim 0$, $\mu_2 \gg \tilde{\mu}_1$. Representing v_1 as a function of v_2 and expressing ξ in terms of v_2 from Eq. (3.3), after several simple transformations we obtain for small $\tilde{\mu}_1$,

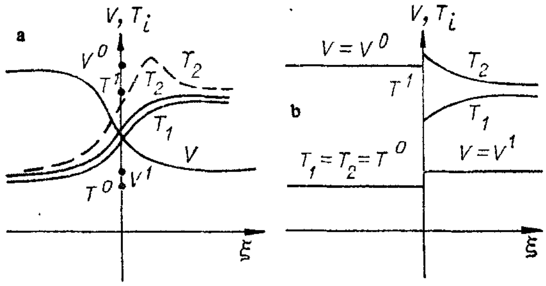

Fig. 1

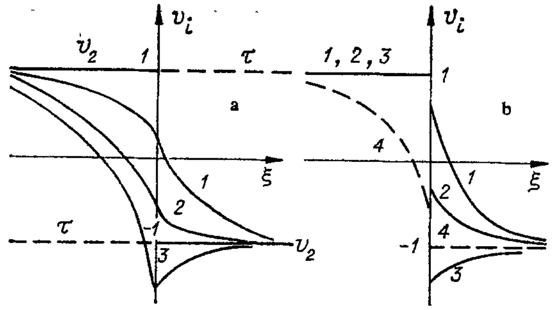

Fig. 2

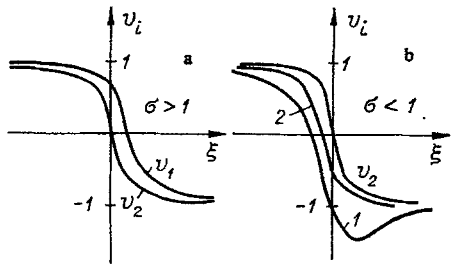

Fig. 3

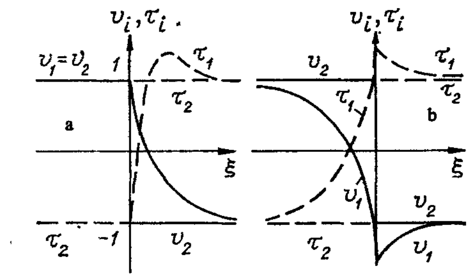

Fig. 4

$$v_1 = v_2 + 2\left(1 - v_2^2\right)\frac{\gamma(\sigma-1)}{\gamma\sigma-1}\int\limits_0^1 \frac{y^{\alpha\beta}dy}{[y(1-v_2)+1+v_2]^2} \quad \text{for} \quad \gamma\sigma > 1, \tag{3.6}$$

$$v_1 = v_2 - 2\left(1 - v_2^2\right)\frac{\gamma(\sigma-1)}{\gamma\sigma-1}\int\limits_0^1 \frac{y^{-\alpha\beta}dy}{[y(1+v_2)+1-v_2]^2} \quad \text{for} \quad \gamma\sigma < 1,$$

where $\alpha = 2\gamma\mu_2/((\gamma+1)^2\varepsilon)$, $\beta = \gamma\tilde{K}/(\gamma\sigma - 1)$. The functions v_1 and v_2 are then related by the equation

$$\frac{dv_1}{dv_2} = \frac{(\gamma-1)\left(v_2^2-1\right) + 2\alpha\beta(\gamma\sigma-1)(v_2-v_1)}{(\gamma\sigma-1)\left(v_2^2-1\right)}. \tag{3.7}$$

Results of a qualitative study of the integral curves of Eq. (3.7) in the plane (v_1, v_2) with consideration of Eqs. (3.3), (3.6) are shown in Fig. 3, with curves 1, 2 corresponding to the function $v_1(\xi)$; 1) for $\alpha\tilde{K}\gamma < \gamma - 1$, $\gamma\sigma < 1$ or $\alpha\tilde{K}\gamma < \gamma\sigma - 1$, $1 < \gamma\sigma < 2\gamma$, and 2) for those inequalities not satisfied.

Thus, at a low concentration of the light component ($\sigma < 1$) nonmonotonic behavior of the light component velocity is possible.

4. We will consider the shock wave structure for a low concentration of one of the mixture components, without assuming strong velocity or temperature interaction. As in Section 3, we let $m_1^0 \sim 0$, $\alpha_1^0 \sim m_1^0$, $\mu_1 \sim m_1^0$ or $\mu_1 \ll m_1^0$, $K \sim m_1^0$, $q \sim m_1^0$. Considering these assumptions in the zeroth approximation in m_1^0 we reduce system (1.4) to the form

$$V_2 + \frac{T_2}{V_2} = 1 + \mu_2\frac{dV_2}{d\xi}, \quad \frac{\gamma}{\gamma-1}T_2 + \frac{V_2^2}{2} = A + \mu_2 V_2\frac{dV_2}{d\xi}, \tag{4.1}$$

$$\sigma\frac{dV_1}{d\xi} + \frac{d(T_1/V_1)}{d\xi} = \tilde{K}(V_2 - V_1) + \tilde{\mu}_1\frac{d^2V_1}{d\xi^2},$$

$$\frac{1}{\gamma-1}\frac{dT_1}{d\xi} + \frac{T_1}{V_1}\frac{dV_1}{d\xi} = \tilde{K}(V_2 - V_1)^2\varkappa_1 + \tilde{q}(T_2 - T_1) + \tilde{\mu}_1\left(\frac{dV_1}{d\xi}\right)^2,$$

where $\tilde{q} = q/m_1^0$, $\tilde{K} = K/m_1^0$, $\tilde{\mu}_1 = \mu_1/m_1^0$.

The first two equations of system (4.1) have the same form as Eq. (3.1), and thus can be integrated explicitly. We will find a solution for the last two equations of system (4.1) for the case of weak shock waves. We transform to new dimensionless velocities and temperatures with Eq. (3.2). Substituting Eq. (3.2) in Eq. (4.1) and dropping terms of higher order in ε, in the zeroth approximation we obtain

$$v_2 = \frac{1 - e^{\alpha\xi}}{1 + e^{\alpha\xi}}, \quad \tau_2 = -v_2, \quad \alpha = \frac{(\gamma+1)^2\varepsilon}{2\gamma\mu_2}, \tag{4.2}$$

$$\sigma\frac{dv_1}{d\xi} + \frac{1}{\gamma}\left((\gamma-1)\frac{d\tau_1}{d\xi} - \frac{dv_1}{d\xi}\right) = \tilde{K}(v_2 - v_1) + \tilde{\mu}_1\frac{d^2v_1}{d\xi^2},$$

$$\frac{d\tau_1}{d\xi} + \frac{dv_1}{d\xi} = -\tilde{q}(\gamma-1)(v_2+\tau_1).$$

Integrating the last two equations of Eq. (4.2) with consideration of boundary conditions (1.5) we have

$$v_1 = \frac{\nu_3(\gamma\tilde{K} + \tilde{q}(\gamma-1)^2) + \gamma\tilde{K}\tilde{q}(\gamma-1)}{\gamma\tilde{\mu}_1(\nu_3-\nu_1)(\nu_3-\nu_2)} e^{\nu_3\xi}\int_{\xi}^{+\infty} e^{-\nu_3 t}v_2\,dt - \tag{4.3}$$

$$- \frac{\nu_2(\gamma\tilde{K} + \tilde{q}(\gamma-1)^2) + \gamma\tilde{K}\tilde{q}(\gamma-1)}{\gamma\tilde{\mu}_1(\nu_2-\nu_1)(\nu_2-\nu_3)} e^{\nu_2\xi}\int_{-\infty}^{\xi} e^{-\nu_2 t}v_2\,dt -$$

$$- \frac{\nu_1(\gamma\tilde{K} + \tilde{q}(\gamma-1)^2) + \gamma\tilde{K}\tilde{q}(\gamma-1)}{\gamma\tilde{\mu}_1(\nu_1-\nu_2)(\nu_1-\nu_3)} e^{\nu_1\xi}\int_{-\infty}^{\xi} e^{-\nu_1 t}v_2\,dt,$$

$$\tau_1 = -v_1 + \tilde{q}(\gamma-1)e^{-\tilde{q}(\gamma-1)\xi}\int_{-\infty}^{\xi}(v_1 - v_2)e^{\tilde{q}(\gamma-1)t}\,dt,$$

where $\nu_1 < \nu_2 < 0 < \nu_3$ are roots of the equation

$$y(\nu) = \nu^3 + \left[(\gamma-1)\tilde{q} - \frac{\sigma-1}{\tilde{\mu}_1}\right]\nu^2 - \left[\frac{\tilde{K}}{\tilde{\mu}_1} + \tilde{q}\frac{(\gamma\sigma-1)(\gamma-1)}{\gamma\tilde{\mu}_1}\right]\nu - \frac{(\gamma-1)\tilde{K}\tilde{q}}{\tilde{\mu}_1} = 0.$$

The order in which these roots are located follows from the inequalities:
$y(\nu = -(\gamma-1)\tilde{q}) > 0$, $y(0) < 0$, $y(+\infty) > 0$. The expressions for v_1 and τ_1 are of complex form and in the general case may be found numerically or expressed in terms of hypergeometric functions. Taking the limit $\mu_i \to 0$ ($i = 1, 2$) in Eq. (4.2), we obtain

$$v_2(\xi) = \begin{cases} 1, & \xi < 0, \\ 0, & \xi = 0, \\ -1, & \xi > 0, \end{cases} \quad \tau_2 = -v_2.$$

If $\sigma > 1$, then

$$v_1(\xi) = \begin{cases} 1, \quad \xi < 0, \\ -1 + \dfrac{2}{(1-\sigma)(\nu_1-\nu_2)}\left[\left(\dfrac{\tilde{K}\tilde{q}(\gamma-1)}{\nu_1} + \tilde{K} + \tilde{q}\dfrac{(\gamma-1)^2}{\gamma}\right)e^{\nu_1\xi} - \right. \\ \qquad \left. - \left(\dfrac{\tilde{K}\tilde{q}(\gamma-1)}{\nu_2} + \tilde{K} + \tilde{q}\dfrac{(\gamma-1)^2}{\gamma}\right)e^{\nu_2\xi}\right], \quad \xi > 0, \end{cases}$$

where $\nu_1 < \nu_2 < 0$ are the roots of the equation

$$(\sigma-1)\nu^2 + \left[\tilde{K} + \tilde{q}(\gamma\sigma-1)\frac{\gamma-1}{\gamma}\right]\nu + (\gamma-1)\tilde{K}\tilde{q} = 0. \tag{4.4}$$

If $\sigma < 1$, then

$$v_1(\xi) = \begin{cases} 1 - \frac{2}{(\nu_3 - \nu_2)(1-\sigma)}\left[\frac{\tilde{K}\tilde{q}(\gamma-1)}{\nu_3} + \tilde{K} + \frac{\tilde{q}(\gamma-1)^2}{\gamma}\right]e^{\nu_3\xi}, & \xi < 0, \\ -1 + \frac{2}{(\nu_2 - \nu_3)(1-\sigma)}\left[\frac{\tilde{K}\tilde{q}(\gamma-1)}{\nu_2} + \tilde{K} + \frac{\tilde{q}(\gamma-1)^2}{\gamma}\right]e^{\nu_2\xi}, & \xi > 0, \end{cases}$$

$$\tau_1(\xi) = \begin{cases} -1 + \frac{\nu_3}{\nu_3 + \tilde{q}(\gamma-1)}(1 - v_1), & \xi < 0, \\ 1 - \frac{\nu_2}{\nu_2 + \tilde{q}(\gamma-1)}(1 + v_1), & \xi > 0, \end{cases}$$

where $\nu_2 < 0$ or $\nu_3 > 0$ are roots of Eq. (4.4) at $\sigma < 1$.

The qualitative behavior of the functions $v_i(\xi)$, $\tau_i(\xi)$ as $\mu_i \to 0$ is shown in Fig. 4a, b for the cases $\sigma > 1$ and $\sigma < 1$ respectively. Thus, in the case of a low concentration of the heavy component (Fig. 4a) the mixture is in equilibrium right up to the shock wave front, after which the velocity and temperature of the heavy component relax to equilibrium values with no discontinuity. The heavy component temperature distribution in the shock wave is nonmonotonic. With a low concentration of the light component the mixture on both sides of the shock transition is in nonequilibrium conditions, tending to equilibrium as $\xi \to \pm\infty$. The temperature and velocity profiles of the light component are nonmonotonic.

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SPALL DAMAGE TO A LIQUID METAL ACCOMPANYING PULSED ACTION OF RADIATION

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The study of spallation accompanying the appearance of short-duration tensile stresses in a material, exceeding the material's tensile strength, is widely used to investigate the dynamic strength of solids [1]. Such stresses appear, in particular, in the presence of thermal shocks — pulsed volume liberation of energy in a material accompanying pulse durations t_p satisfying the condition $t_p \lesssim l/c$, where l is the characteristic size of the region of energy liberation and c is the velocity of sound in the material. As shown in [2], instantaneous thermal shocks (corresponding to the more stringent condition $t_p \ll l/c$), can lead to spalls with energy inputs significantly lower than the heat of fusion and, especially, the heat of evaporation of the material. In experiments modeling thermal shocks, laser radiation is usually used as the course of energy liberation for weakly absorbing media and relativistic electron beams (REB) are used for metals [3, 4]. Experiments with REB correspond, as a rule, to the weaker condition $t_p \lesssim l/c$.

Negative stresses and spalls can be observed not only in solids, but also in liquid metals [4]. The possibility of spalls must be taken into account, in particular, in setting up liquid-metal shielding of the first wall of pulsed thermonuclear reactors [5]. The action of fluxes of charged particles and x-ray radiation on a liquid metal usually leads to strong

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